Question 1

The control system shown in Figure 1 is designed to maintain the exit temperature, T_2, from a tank at a set-point of T_{sp}, by manipulating the heat input rate, $\dot{Q}$, supplied to a heat transfer coil. The insulated tank may be assumed to be well-mixed and has a constant volume, V. The liquid feed has a constant volumetric flow rate, q, but is subject to disturbances in the feed temperature, T_1.

(a) By deriving an input-output model for the tank, show that;

$$\bar{T}_2(s) = \left(\frac{1}{1+\tau s}\right)\bar{T}_1(s) + \left(\frac{K_p}{1+\tau s}\right)\bar{\dot{Q}}(s)$$

and obtain expressions for the parameters K_p and τ.

[4 marks]

(b) The system uses a PI controller with a transfer function

$$G_c(s) = K_c\left(1 + \frac{1}{\tau_I s}\right)$$

where K_c is the controller gain and τ_I the integral reset time. Show, using Figure 2, that the closed loop transfer function **for a disturbance in the feed temperature,** T_1, is of the form

$$\frac{\bar{T}_2}{\bar{T}_1} = \frac{G_D}{1 + G_C G_P}$$

giving expressions for G_D and G_P.

[5 marks]

(c) Find the conditions under which the response is (i) underdamped and (ii) overdamped. For each case, sketch the variation in T_2 with time, for a step change in feed temperature T_1.

[7 marks]

(d) Initially the system is at steady-state. For the data given below, calculate the maximum deviation of the exit temperature from its set point, when the feed undergoes a step change of 10 K. Comment on the response of the control system and how it might be improved.

[4 marks]

Data:

q = 0.005 $m^3 s^{-1}$, V = 0.5 m^3, K_c = 180 kW/K, τ_I = 36 s.
Liquid density = 1000 kgm^{-3}.
Specific Heat Capacity = 4 kJ/kg K.

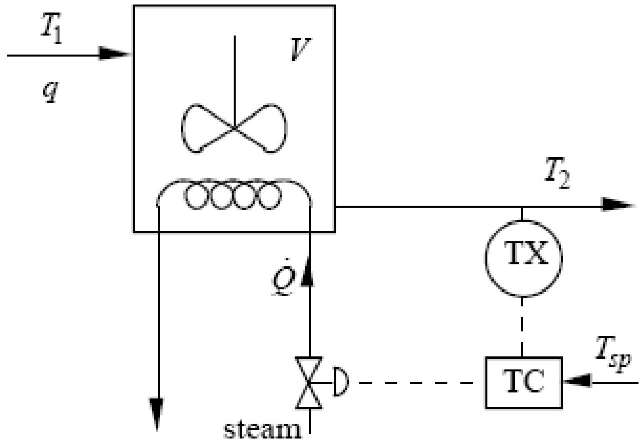

Figure 1: Schematic of Control System.

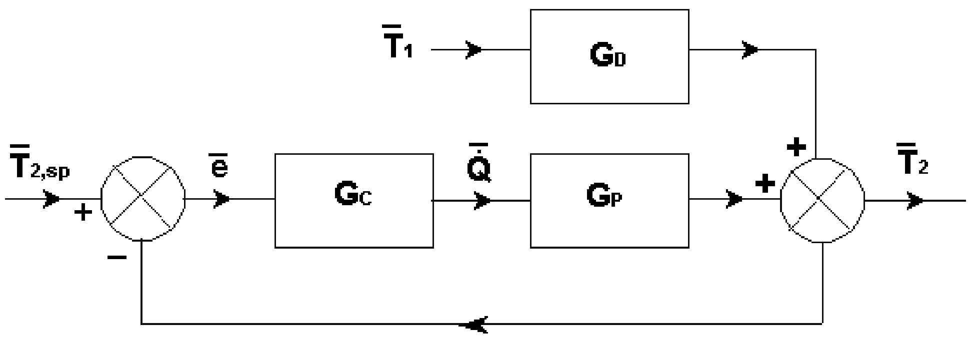

Figure 2: Block Diagram of the Closed Loop System.